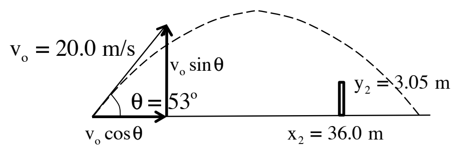
Problem 4.17

A sketch of the situation (always a good idea if you have the time) is presented to the right.



a.) Probably the easiest way to do this to determine how long it takes for the ball to go the horizontal distance, then calculate its y-coordinate at that point in time. Thusly:

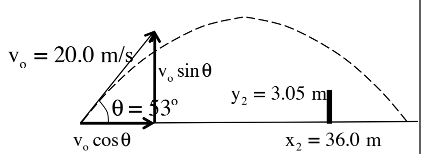
$$
\begin{aligned}
x_2 &= \cancelto{0}{x_1} + (v_o\cos\theta)\Delta t + \frac{1}{2}\cancelto{0}{a_x}(\Delta t)^2 \\
\Rightarrow\quad (36.0\text{ m}) &= (20.0\text{ m/s})(\cos 53^\circ)\Delta t \\
\Rightarrow\quad \Delta t &= 2.99\text{ s}
\end{aligned}
$$

Equation A

In the y-direction:

$$
\begin{aligned}
y_2 &= \cancelto{0}{y_1} + (v_o\sin\theta)\Delta t + \frac{1}{2}(-g)(\Delta t)^2 \\
\Rightarrow\quad y_2 &= (20.0\text{ m/s})(\sin 53^\circ)(2.99\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(2.99\text{ s})^2 \\
&= 3.94\text{ m}
\end{aligned}
$$

In other words, it takes 2.99 seconds for the ball to get to the goal post, and when it gets there it's height above the ground is 3.94 meters, which is plenty to make it over the 3.05 meter post height.



b.) Is it moving upward or downward at the goal post?

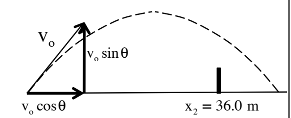
We can determine the time it takes for the ball to get to the top of its arc using:

$$
\begin{aligned}
(\cancelto{0}{v_{top,y}}) &= (v_{o,y}) + a_y\Delta t_{halfway} \\
\Rightarrow\quad (0) &= (20.0\text{ m/s})(\sin 53^\circ) + (-9.80\text{ m/s}^2)\Delta t_{halfway} \\
\Rightarrow\quad \Delta t_{halfway} &= 1.63\text{ s}
\end{aligned}
$$

If the time to the halfway point was greater than the 2.99 seconds required to get to the goal post, the ball would be moving upward as it passed over the post. That time is less than 2.99 seconds, which means the ball has reached its maximum height and is now moving downward.

Sooo, it makes it and is moving downward!

A more exotic way to do this is to generate a function for the body's *y-position* as a function of time, take its time derivate *dy/dt* and see if, at the time it takes for the ball to get to the goal, that derivative is positive (*y* getting bigger meaning the body is elevating) or negative (y getting smaller meaning the body is dropping). Trying that, we get:



$$
\begin{aligned}
\cancelto{0}{y} &= y_{initial} + v_{1,y}(\Delta t) + \frac{1}{2}a_y(\Delta t)^2 \\
&= 0 + (v_o\sin\theta)t + \frac{1}{2}(-g)t^2 \\
\Rightarrow\quad y &= (v_o\sin\theta)t + \frac{1}{2}(-g)t^2
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{dy}{dt} &= \frac{d\left[(v_o\sin\theta)t + \frac{1}{2}(-g)t^2\right]}{dt} \\
&= (v_o\sin\theta) - \frac{1}{2}(g)(2t) \\
&= v_o\sin\theta - gt
\end{aligned}
$$

This isn't surprising. It's just another kinematic equation.

In *Part a*, we determine the ball's flight time to the goal post at x = 36.0 meters. It was 2.99 seconds. With that, we can take the derivative and evaluate it at t = 2.99 second. Doing so yields:

$$
\begin{aligned}
\frac{dy}{dt} &= \frac{d\left[(v_o\sin\theta)t + \frac{1}{2}(-g)t^2\right]}{dt} \\
&= (v_o\sin\theta) - \frac{1}{\cancel{2}}(g)(\cancel{2}t) \\
&= (20\text{ m/s})\sin 53^\circ - (9.80\text{ m/s}^2)(2.99\text{ s}) \\
&= -13.3\text{ m/s}
\end{aligned}
$$

Apparently, at this time, *y* is diminishing (that's what a negative slope means) so again, apparently, the ball is coming down as it passes over the goal post.